# Modal Cross Power in Quasi-TEM Transmission Lines

Dylan F. Williams, *Senior Member, IEEE*, and Frank Olyslager, *Member, IEEE*

***Abstract*—This letter examines modal cross power in electromagnetic transmission lines. It shows that the cross powers of nearly degenerate modes may be large in quasi-TEM multiconductor transmission lines typical of modern electronic circuits at moderate and low microwave frequencies. The letter develops simple expressions to estimate the magnitude of these cross powers from the "power-normalized" conductor impedance and admittance matrices of the lines.**

## I. INTRODUCTION

THIS LETTER examines modal cross power in multiconductor transmission lines typical of modern electronic circuits and presents expressions useful for estimating their importance.

The total electric field $\boldsymbol{E}$ and magnetic field $\boldsymbol{H}$ in a closed transmission line uniform in $z$ and constructed of linear isotropic materials can be written as

$$\boldsymbol{E} = \sum_n c_n^{\pm} e^{\pm\gamma_n z}(\boldsymbol{e}_{tn} \pm e_{zn}\boldsymbol{z}) \tag{1}$$

and

$$\boldsymbol{H} = \sum_n c_n^{\pm} e^{\pm\gamma_n z}(\pm\boldsymbol{h}_{tn} + h_{zn}\boldsymbol{z}) \tag{2}$$

where $c_n^{\pm}$ are the forward and reverse excitation coefficients of the $n$th mode, $\gamma_n$ is its propagation constant, and its transverse modal electric and magnetic fields $\boldsymbol{e}_{tn}$ and $\boldsymbol{h}_{tn}$ and its longitudinal modal electric and magnetic fields $e_{zn}$ and $h_{zn}$ are functions only of the transverse coordinates $x$ and $y$ [1]. Here $\boldsymbol{z}$ is the unit vector in the $z$ direction, which coincides with the direction of propagation, and the time harmonic dependence $e^{+j\omega t}$, where $\omega$ is the real angular frequency, has been suppressed. In open guides we must add a continuous spectrum of modes to this discrete set [2].

When only a finite number of the discrete modes are excited in the line, the total complex power $p$ is

$$p = \int \boldsymbol{E} \times \boldsymbol{H}^* \cdot \boldsymbol{z}\, dS = \sum_{nm} (c_n^+ e^{\gamma_n z} + c_n^- e^{-\gamma_n z}) (c_m^+ e^{\gamma_m z} - c_m^- e^{-\gamma_m z})^* P_{nm} \tag{3}$$

where the sum is taken over all the excited modes, $P_{nm} \equiv \int \boldsymbol{e}_{tn} \times \boldsymbol{h}_{tm}^* \cdot \boldsymbol{z}\, dS$, and the integrals are performed over the transmission-line cross section. We will call the $P_{nm}$ for $n \neq m$ the modal cross powers and will use the unitless scalars $\zeta_{nm} \equiv P_{nm}P_{mn}/(P_{nn}P_{mm})$ to quantify their significance.

Lossless modes are power orthogonal when they are not degenerate; that is, their modal cross powers are zero ($\zeta_{nm} = 0$) when $\gamma_n^2 \neq \gamma_m^2$ [1]. Most equivalent circuit theories for multimode transmission lines begin with assumptions of power-orthogonal modes.

When $\zeta_{nm}$ is nonzero, which is only possible in lossy guides, the total power in the line can no longer be calculated as a simple sum of the powers carried by each pair of forward and backward modes and, in the terminology of [3], we would say that the modes are coupled. In these cases equivalent-circuit theories for multimode transmission lines based on assumptions of power-orthogonal modes would not apply.

Modal symmetries eliminate the cross powers of the modes of low-loss circular and coaxial waveguides [3]. The cross powers of low-loss rectangular waveguide modes are generally small except at frequencies where the modes are nearly degenerate. At these frequencies the modes couple and the field patterns of each of the lossy coupled modes can be represented to first order as linear combinations of the field patterns of lossless uncoupled modal solutions, which gives rise to large modal cross powers [3], [4]. While [3] and [4] use perturbation theories to construct the actual modal fields from superpositions of lossless solutions, this theory cannot be applied to highly lossy lines typical of modern circuits. In any case, since these near degeneracies in rectangular waveguides are limited to narrow bands of frequencies above the conventional upper frequency limit of the guide, they may often be ignored in practice.

Reference [5] remarked that the cross powers of the two dominant quasi-TEM modes of the multiconductor transmission line structure of Fig. 1 are large at useful frequencies and illustrated the importance of accounting for them in thermal noise calculations. Faché and De Zutter have constructed an equivalent circuit theory based on power-normalized "conductor" voltages and currents that accounts rigorously for modal cross powers even when losses are large [6]. This theory has been clarified and extended in [7]–[9]. These works do not, however, discuss the mechanisms and conditions that give rise to large modal cross powers.

The high resistive losses of the small printed multiconductor transmission lines typical of modern electronic circuits complicate their modal dispersion relations and often create near degeneracies over broad ranges of useful frequencies. In this work we will investigate the cross powers of the modes

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D. F. Williams is with the National Institute of Standards and Technology, Boulder, CO 80303 USA.

F. Olyslager is with the Belgian National Fund for Scientific Research, Department of Information Technology, University of Ghent, Sint-Pietersnieuwstraat 41, B-9000 Ghent, Belgium.

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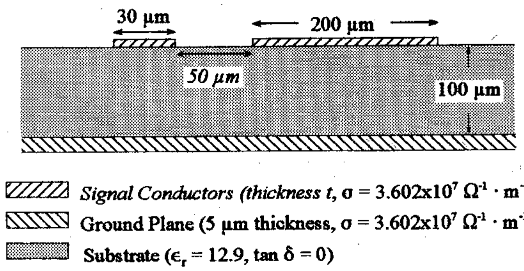


Fig. 1. The coupled microstrip transmission lines of [5].

of some typical lossy multiconductor transmission lines and show that these near degeneracies result in large $\zeta_{nm}$. We will illustrate this with the coupled asymmetric microstrip lines of Fig. 1 and will develop useful expressions for $\zeta_{nm}$ in terms of the "power-normalized" transmission-line impedances and admittances per unit length of [6], which may often be estimated from static analyzes [10].

## II. Quasi-TEM Illustration

The coupled lines of Fig. 1 support two dominant quasi-TEM modes, which are commonly called the $c$ and $\pi$ modes, and which correspond to the even and the odd mode of the symmetric case, respectively. We verified that these modes were quasi-TEM by calculating the ratios $\int |e_z|^2\, dS / \int |\boldsymbol{e}_t|^2\, dS$ and $\int |h_z|^2\, dS / \int |\boldsymbol{h}_t|^2\, dS$ with the full-wave method of [11] and found that they approached zero at low frequencies and were less than 0.001 below 10 GHz.

At high frequency, the metal losses in the coupled lines of Fig. 1 can be neglected and the propagation constant of the $c$ mode, which concentrates energy in the dielectric substrate, is substantially larger than that of the $\pi$ mode (which has significant energy in the air region above the dielectric substrate). The higher loss of the $\pi$ mode, however, forces its propagation constant to rise more rapidly at low frequencies than the propagation constant of the $c$ mode, inevitably causing $\gamma_c$ and $\gamma_\pi$ to become nearly degenerate at some intermediate frequencies.

Fig. 2 plots the square root of $\zeta_{c\pi}$ calculated directly from the fields determined by the full-wave method of [11] in solid lines, values that we verified with a method based on that of [12]. Although $\zeta_{c\pi}$ is always zero due to the even/odd symmetry of the fields when the conductor widths are equal, $\zeta_{c\pi}$ for the asymmetric case shown in the figure rises when $\gamma_c$ and $\gamma_\pi$ become nearly degenerate [5], an observation consistent with similar phenomena observed in rectangular waveguides [3], [4]. For the line of Fig. 1 with 0.5-$\mu$m-thick conductor metal, for example, $\gamma_c$ and $\gamma_\pi$ become close in the frequency range 300 MHz–5 GHz, while $\zeta_{c\pi}$ peaks at about 1 GHz.

## III. Algebraic Expressions for $\zeta_{nm}$

The $P_{nm}$ fix relations between the modal and the power-normalized "circuit" voltages and currents of [6] and can be determined from products of the matrices relating those quantities. The unitless measure $\zeta_{nm}$ can be determined solely from the matrices of power-normalized conductor impedances

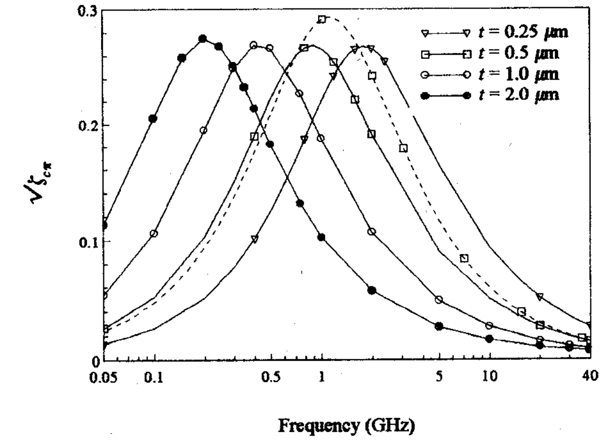


Fig. 2. The square root of $\zeta_{c\pi}$ for the asymmetric coupled microstrip lines of Fig. 1. The solid lines correspond to values calculated directly from modal electromagnetic fields determined by the full-wave method of [11]. The dashed line correspond to values for $t = 0.5\,\mu\text{m}$ calculated from (2) and estimates of the transmission-line circuit parameters.

per unit length $\underline{Z} \equiv \underline{R} + j\omega \underline{L}$ and admittances per unit length $\underline{Y} \equiv \underline{G} + j\omega \underline{C}$ of the line without detailed knowledge of how the modal and circuit quantities in the theory of [6] are normalized. $\zeta_{nm}$ is found from $\underline{Z}$ and $\underline{Y}$ by

$$\zeta_{nm} = \frac{[b(\lambda_m)^\dagger a(\lambda_n)][b(\lambda_n)^\dagger a(\lambda_m)]}{[b(\lambda_n)^\dagger a(\lambda_n)][b(\lambda_m)^\dagger a(\lambda_m)]} \tag{4}$$

where superscript "$\dagger$" signifies Hermitian conjugate (conjugate transpose), $a(\lambda_n)$ and $a(\lambda_m)$ are the eigenvectors of $\underline{\beta} \equiv \underline{Y}\,\underline{Z}$ with eigenvalues $\lambda_n = \gamma_n^2$ and $\gamma_m = \gamma_m^2$, and $b(\lambda_n)$ and $b(\lambda_m)$ are the eigenvectors of $\underline{\alpha} \equiv \underline{Z}\,\underline{Y}$ with eigenvalues $\lambda_n$ and $\lambda_m$ [6].

Reference [9] shows for the $c$ and $\pi$ modes of Fig. 1 that $\underline{G}$ is nearly zero when there are no dielectric losses, that $\underline{C}$ is nearly constant with frequency, and that $\underline{L}$ rises only slightly at the low frequencies while $\underline{R}$ increases only moderately at the high frequencies in a fashion consistent with the effects of field-penetration into the thin metal conductors, making them easy to estimate. Fig. 2 shows in dashed lines $\zeta_{c\pi}$ for the metal thickness $t = 0.5\,\mu\text{m}$ calculated from (4) using static estimates of $\underline{L}$ and $\underline{C}$ from the method of [10], $\underline{G} = 0$, and the low-frequency limit of $\underline{R}$, which we determined from the dc resistances of the conductors. It compares it to the direct calculation from the modal electromagnetic fields determined by the full-wave method of [11] (solid lines) and shows that the estimate is accurate enough to determine when the modal cross powers are significant. We found similar agreement for the other metal thicknesses of Fig. 2.

When $\underline{Z}$ and $\underline{Y}$ are diagonal, then (4) shows that $\zeta_{nm} = 0$, as $\underline{\alpha} = \underline{\beta}$ are also diagonal and their eigenvectors can be taken to be the columns of the identity matrix.

When $\underline{Z}$ and $\underline{Y}$ are symmetric, which we found to be a very good approximation for the $c$ and $\pi$ modes of Fig. 1 and which [13] argues is true for all quasi-TEM modes, then $\underline{\beta} = \underline{\alpha}^t$, where superscript "$t$" signifies transpose. This implies that $b(\lambda_m)^t a(\lambda_n) = b(\lambda_n)^t a(\lambda_m) = 0$, and we see from (4) that $\zeta_{nm} = 0$ whenever the eigenvectors of $\underline{\alpha}$ and $\underline{\beta}$ can be taken real.

Since for the $c$ and $\pi$ modes of Fig. 1 $\underline{G}$ is nearly zero and $\underline{C}$, $\underline{L}$, and $\underline{R}$ depend only weakly on frequency [9], $\alpha$ becomes purely real at very high frequencies ($\omega|L_{ij}| \gg |R_{ij}|$) and purely imaginary at very low frequencies ($\omega|L_{ij}| \ll |R_{ij}|$). $\underline{Z}$ and $\underline{Y}$ are positive definite, which [13] argues is always true for quasi-TEM modes, with the consequence that the eigenvectors of $\underline{\alpha}$ are also nearly real at the two frequency extremes. This explains the tendency seen in Fig. 2 of $\zeta_{c\pi}$ to approach zero at these extremes.

If $\underline{G} = 0$ and $\underline{R}, \underline{L}$, and $\underline{C}$ are independent of frequency, which are reasonable approximations for the case studied here [9], scaling $\underline{R}$ and $\omega$ by a constant real factor $s$ scales $\underline{\alpha}$ by $s^2$. This leaves the eigenvectors of $\underline{\alpha}$, and thus the value of $\zeta_{nm}$, constant and explains the shift of the maximum of $\zeta_{c\pi}$ in Fig. 2 to lower frequencies when the conductor losses are reduced by increasing metal thickness. It also explains why the maximum value and shape of $\zeta_{c\pi}$ does not change greatly as the metal thickness is varied.

For two modes the eigenvalue/eigenvector problem can be solved explicitly in terms of the elements of $\underline{\alpha}$ and $\underline{\beta}$. When $\underline{\beta} = \underline{\alpha}^t$ then (4) becomes

$$\zeta_{12} = 1 - \frac{|q(\lambda_1) - q(\lambda_2)|^2}{|q(\lambda_1) - q(\lambda_2)^*|^2} \quad (5)$$

where $q(\lambda) \equiv (\lambda - \alpha_{11})/\alpha_{12} = \alpha_{21}/(\lambda - \alpha_{22})$ is the ratio of the second to the first element of the eigenvector associated with the eigenvalue $\lambda = \frac{1}{2}(\alpha_{11} + \alpha_{22}) \pm \frac{1}{2}\sqrt{(\alpha_{11} - \alpha_{22})^2 + 4\alpha_{21}\alpha_{12}}$. Equation (5) shows that $\zeta_{12}$ is real and less than or equal to one.

An equivalent form for (5) is

$$\zeta_{12} = 1 - \frac{|\lambda_1 - \lambda_2|^2}{|\mathrm{Im}(\rho\sigma)|^2 + |\mathrm{Re}((\lambda_1 - \lambda_2)\sigma)|^2} \quad (6)$$

where $\rho = \pm(\alpha_{11} - \alpha_{22})$ and $\sigma = \pm\alpha_{12}^*/|\alpha_{12}| = \pm|\alpha_{12}|/\alpha_{12}$ or $\sigma = \pm\alpha_{21}^*/|\alpha_{21}| = \pm|\alpha_{21}|/\alpha_{21}$. $\mathrm{Im}(\rho\sigma)$ will be small when losses are low, so $\zeta_{12}$ may remain small even quite near the degeneracies of low-loss modes. When $\mathrm{Im}(\rho\sigma)$ differs significantly from zero, which will usually be the case in lossy structures, (6) shows that $\zeta_{12}$ will approach one as two modes become degenerate. This shows that nearly degenerate lossy modes will often have large cross powers and explains the observed rise of $\zeta_{c\pi}$ in Fig. 2 where $\gamma_c$ and $\gamma_\pi$ were close.

## IV. CONCLUSION

Both full-wave field calculations and static estimates show that large modal cross powers are not limited to exotic or highly lossy structures, but occur between nearly degenerate modes of practical planar quasi-TEM multiconductor transmission lines in common use in modern electronic circuits. The cross-power levels can be determined from the power-normalized equivalent-circuit parameters of the transmission line, which have a weak dependance on frequency and are easily estimated. The results show that the modal description can have a complicated dependance on frequency even when the equivalent-circuit description does not and argue that equivalent circuit theories such as those described in [6]–[9], which rigorously account for modal cross powers, are required to treat these common circuit elements.

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